Factorising by completing the square

When we can't factorise by finding factors of 'c' or 'ac' that add to 'b' in a trinomial, then we use the method called completing the square.

eg. $x^2 + 6x - 3$ (no factors of -3 add to $+6$)

so look for a squared expression that gives the first two terms.

eg $(x+3)^2$

expand $= \underline{x^2 + 6x} + 9$

but this is $+9$ too big so we need to -9 from squared expression

$\rightarrow (x+3)^2 - 9$

$= ((x+3)^2 - 9) - 3$

simplify

$= (x+3)^2 - 12$

We can factorise further using DOTS

where $a^2 = (x+3)^2$

& $b^2 = 12$

which gives

$(a+b)(a-b) = a^2 - b^2$

$$\underbrace{(x+3+\sqrt{12})}(x+3-\sqrt{12}) = (x+3)^2 - 12$$

this is how the book writes the answers.

Remember you can always expand your factorised answer to check.

eg $(x+3)^2 - 12$

$= x^2 + 6x + 9 - 12$

$= x^2 + 6x - 3$ ✓ matches initial expression

Factorising by completing the square cont.

eg $\quad x^2 + 10x + 5$

rewrite/factorise in form of $a(x+b)^2 + c$ where $b = \frac{1}{2}$ coefficient of x.

$$= \left((x+5)^2 - 25\right) + 5$$

$$= (x+5)^2 - 20$$

$$\underbrace{x^2 + 10x}_{} + 5$$

$$\left(x + \frac{10}{2}\right)^2 = x^2 + 10x + 25$$

$+25$ too much so -25.

DOTS

$$\left(x + 5 + \sqrt{20}\right)\left(x + 5 - \sqrt{20}\right)$$

Be careful with odd coefficients of x - fractions become involved!

eg $\quad x^2 - 7x - 2$

$$\underbrace{x^2 - 7x}_{}$$

$$\left(-\frac{7}{2}\right)^2 = \frac{7^2}{2^2} = \frac{49}{4}$$

$$= \left(\left(x - \frac{7}{2}\right)^2 - \frac{49}{4}\right) - 2$$

$$\left(x - \frac{7}{2}\right)^2 = x^2 - 7x + \frac{49}{4}$$

rewrite as equivalent fraction $\left(\times\frac{4}{4}\right)$

$+\frac{49}{4}$ too much so $-\frac{49}{4}$

$$= \left(x - \frac{7}{2}\right)^2 - \frac{49}{4} - \frac{8}{4}$$

$$= \left(x - \frac{7}{2}\right)^2 - \frac{57}{4}$$

DOTS. $\quad \left(x - \frac{7}{2} + \sqrt{\frac{57}{4}}\right)\left(x - \frac{7}{2} - \sqrt{\frac{57}{4}}\right)$

but $\sqrt{\frac{57}{4}} = \frac{\sqrt{57}}{2}$.

$$= \left(x - \frac{7}{2} + \frac{\sqrt{57}}{2}\right)\left(x - \frac{7}{2} - \frac{\sqrt{57}}{2}\right)$$